

noncommutative quantum electrodynamics

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Abstract

We show that a simple and reasonable generalization of the anomalous interaction between the neutral pion and two photons can induce the C-violating three photon decay of the neutral pion in noncommutative quantum electrodynamics. We find that it is mandatory for consistency reasons to include simultaneously the normal neutral pion and photon interaction in which the neutral pion transforms under $U(1)$ in a similar way as in the adjoint representation of a non-Abelian gauge theory. We demonstrate that the decay has a characteristic distribution although its rate still seems too small to be experimentally reachable in the near future. We also describe how to manipulate phase space integration correctly when Lorentz invariance is lost.

Keywords: noncommutative field theory, anomalous pion photon interaction, charge conjugation violation, rare pion decay

PACS: 11.15.-q, 11.40.-q, 11.30.Er, 14.40.AQ

Noncommutative field theories have recently received a lot of attention mainly because of their connection to string theories [1]. But Noncommutative field theories are certainly interesting in their own right. A possible way to construct the noncommutative version of a field theory from its ordinary commutative counterpart is by replacing the usual product of fields in the action with the \star -product of fields. The \star -product of the two fields $\phi_1(x)$ and $\phi_2(x)$ is defined as

$$(\phi_1 \star \phi_2)(x) = \left[\exp \left(\frac{i}{2} \theta^{\mu\nu} \partial_\mu^x \partial_\nu^y \right) \phi_1(x) \phi_2(y) \right]_{y=x}, \quad (1)$$

where $\theta^{\mu\nu}$ is a real antisymmetric constant matrix that parameterizes the noncommutativity of spacetime,

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}, \quad (2)$$

and has dimensions of length squared.

The noncommutative quantum electrodynamics (NCQED) of photons is then given by the following Lagrangian [2],

$$\begin{aligned} \mathcal{L}_F &= -\frac{1}{4} F^{\mu\nu} \star F_{\mu\nu}, \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu + ie[A_\mu, A_\nu]_\star, \end{aligned} \quad (3)$$

where the Moyal bracket is defined as

$$[\phi_1, \phi_2]_\star = \phi_1 \star \phi_2 - \phi_2 \star \phi_1. \quad (4)$$

The action $\int d^4x \mathcal{L}_F$ is invariant under the generalized $U(1)$ gauge transformation,

$$A_\mu \rightarrow A'_\mu = U \star A_\mu \star U^{-1} + ie^{-1} U \star \partial_\mu U^{-1}, \quad U(x) = (\exp(i\lambda(x)))_\star, \quad (5)$$

under which $F_{\mu\nu}$ transforms as follows,

$$F_{\mu\nu} \rightarrow F'_{\mu\nu} = U \star F_{\mu\nu} \star U^{-1}. \quad (6)$$

Note that the neutral photon interacts with itself due to the Moyal bracket term in $F_{\mu\nu}$ as in the usual non-Abelian gauge theory. One must then be careful with the gauge fixing procedure since the ghost also interacts as opposed to the usual QED in which it is free,

$$\mathcal{L}_{gf} = -\frac{1}{2\xi} (\partial A)^2 + (\partial^\mu \bar{c}) (\partial_\mu c + ie[A_\mu, c]_\star), \quad (7)$$

where we have freely replaced one of the star products by the usual one because one always has in the action, $\int d^4x \phi_1 \star \phi_2 = \int d^4x \phi_1 \phi_2$. The matter fields can also be incorporated. The interested reader should consult the above references for details.

The phenomenological implications of the NCQED have began to appear very recently. They are roughly classified into two categories, namely the small corrections to precisely measured quantities in low energy atomic systems [3] and the relatively larger corrections to QED processes at future high energy linear colliders [4]. In this work we shall consider a novel combined effect of the NCQED and the generalized axial anomaly in noncommutative spacetime, i.e., the three photon (γ) decay of the neutral pion (π^0). This decay is a C-violating process which proceeds in the standard model through weak interactions and is thus very small. The appearance of multi-photons in the final state suppresses further the decay because gauge invariance demands many factors of the photon momenta whose dimensions have to be balanced by some heavier mass scales. Indeed, an estimate [5] showed its branching ratio in the standard model is of order 10^{-31} , too tiny to be experimentally feasible. This makes the decay a possible testing ground for C violation beyond weak interactions, for example, in electromagnetic or strong interactions. Our basic observations are two fold. The NCQED violates the C symmetry [6] through the Moyal bracket term and this should have some positive effect on the decay $\pi^0 \rightarrow 3\gamma$ because the latter may happen at the electromagnetic strength. On the other hand, since the Lorentz invariance is spoiled by the constant matrix $\theta_{\mu\nu}$ we may have completely different gauge invariant structures for the decay amplitude so that the suppression introduced by gauge invariance may become less severe. If both are true we shall expect a large enhancement of the decay.

The neutral pion decays dominantly into two photons, which is driven by the axial anomaly,

$$\mathcal{L}_A = -J\epsilon^{\mu\nu\rho\sigma}\pi^0 F_{\mu\nu}F_{\rho\sigma}, \quad (8)$$

where $F_{\mu\nu}$ is the usual QED electromagnetic tensor, and the constant $J = \frac{N_c e^2}{96\pi^2 f_\pi}$ with N_c being the color number and f_π the pion decay constant. This term is unique in the Wess-Zumino-Witten action [7] in the sense that it involves the least number of Goldstone bosons while involving the most number of photons. Our argument is that this term should be extended most naively if there is any kind of generalization of the axial anomaly to noncommutative spacetime. Then a simple and reasonable guess is

$$\mathcal{L}_{NCA} = -J\epsilon^{\mu\nu\rho\sigma}\pi^0 F_{\mu\nu} \star F_{\rho\sigma}, \quad (9)$$

where $F_{\mu\nu}$ is now the NCQED electromagnetic tensor appearing in \mathcal{L}_F and once again we have dropped the star of π^0 with F 's. Furthermore, the above term should not be affected

by generalization of other terms in the Wess-Zumino-Witten action due to the same reason of uniqueness. This guess has got some support from recent one loop approaches to the anomaly in noncommutative spacetime [8].

We notice that \mathcal{L}_{NCA} contains the desired $\pi^0 \rightarrow 3\gamma$ transition term. So, together with $\mathcal{L}_F + \mathcal{L}_{gf}$ we might think we were already prepared to calculate the decay amplitude. Actually, for consistency reasons to be explained later on, we are still missing one piece: we have to include simultaneously the normal direct interactions between the photon and π^0 . As shown in Hayakawa's papers in Refs. [2], besides the possibility of being invariant, the neutral particle fields may also undergo a nontrivial transformation under $U(1)$,

$$\pi^0 \rightarrow \pi^{0'} = U \star \pi^0 \star U^{-1}. \quad (10)$$

which resembles the adjoint transformation in the usual non-Abelian gauge theory. Since $F_{\mu\nu}$ itself also transforms under $U(1)$, it seems that we must include also the above one to keep uniformness and completeness among neutral fields. The covariant derivative for π^0 is

$$D_\mu \pi^0 = \partial_\mu \pi^0 + ie[A_\mu, \pi^0]_\star, \quad (11)$$

which transforms similarly to π^0 and becomes trivial in the usual commutative spacetime. Finally, the NCQED Lagrangian for π^0 can be written down,

$$\mathcal{L}_{\pi^0} = \frac{1}{2} D_\mu \pi^0 \star D^\mu \pi^0. \quad (12)$$

With all pieces at hand we can now compute the decay amplitude for $\pi^0(p) \rightarrow \gamma(k_1, \epsilon_{1\alpha}) + \gamma(k_2, \epsilon_{2\beta}) + \gamma(k_3, \epsilon_{3\gamma})$, where p and k_i are incoming and outgoing momenta of the π^0 and photons, ϵ_i are photon polarization vectors with Lorentz indices α, β, γ . The corresponding Feynman diagrams are shown in Fig. 1. Let us first list the relevant Feynman rules to set up our notation. The point for the derivation of them is the recursive use of the following Fourier transformation for the star product of functions,

$$(\phi_1 \star \phi_2)(x) = \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} \tilde{\phi}_1(k_1) \tilde{\phi}_2(k_2) \exp[-i(k_1 + k_2) \cdot x] \exp[-ik_1 \theta k_2 / 2], \quad (13)$$

where $\tilde{\phi}_i(k_i)$ are Fourier transforms of $\phi_i(x)$ and $k_1 \theta k_2 = \theta_{\mu\nu} k_1^\mu k_2^\nu$. In the following list of Feynman rules we use q_i to denote the incoming momenta of photons and ghosts, p_i the incoming momenta of the π^0 's and μ or μ_i the photon Lorentz indices. The Feynman rule for the three photon vertex is

$$+2e \sin(q_1 \theta q_2 / 2) V_3^{\mu_1 \mu_2 \mu_3}(q_1, q_2, q_3), \quad (14)$$

with

$$V_3^{\mu_1\mu_2\mu_3}(q_1, q_2, q_3) = (q_1 - q_2)^{\mu_3} g^{\mu_1\mu_2} + (q_2 - q_3)^{\mu_1} g^{\mu_2\mu_3} + (q_3 - q_1)^{\mu_2} g^{\mu_3\mu_1}. \quad (15)$$

Note that the above vertex actually satisfies the Bose symmetry due to momentum conservation and antisymmetry of $\theta_{\mu\nu}$. The ghost-anti-ghost-photon vertex is

$$+2e q_{2\mu} \sin(q_1\theta q_2/2), \quad (16)$$

where q_2 is the incoming anti-ghost momentum. The vertex for the anomalous $\pi^0 \rightarrow 2\gamma$ transition is modified to be

$$-i8J \epsilon^{\mu_1\mu_2\rho\sigma} q_{1\rho} q_{2\sigma} \cos(q_1\theta q_2/2). \quad (17)$$

The new vertex for the contact $\pi^0 3\gamma$ interaction will appear as part of the contribution to the decay amplitude and will be presented below. The final piece of Feynman rule for the $\gamma 2\pi^0$ vertex is

$$-2e (p_1 - p_2)_\mu \sin(p_1\theta p_2/2), \quad (18)$$

which again satisfies the Bose symmetry with respect to π^0 's.

The separate contributions to the decay amplitude from Fig. 1(a)-(c) can be cast in the form,

$$\begin{aligned} \mathcal{A} &= i16eJ \left(\mathcal{A}_a^{\alpha\beta\gamma} + \mathcal{A}_b^{\alpha\beta\gamma} + \mathcal{A}_c^{\alpha\beta\gamma} \right) \epsilon_{1\alpha}^* \epsilon_{2\beta}^* \epsilon_{3\gamma}^*, \\ \mathcal{A}_a^{\alpha\beta\gamma} &= \epsilon^{\mu\alpha\beta\gamma} (A_1 k_{1\mu} + A_2 k_{2\mu} + A_3 k_{3\mu}), \\ \mathcal{A}_b^{\alpha\beta\gamma} &= \frac{A_1}{2d_1} \epsilon_\rho^{\alpha\mu\nu} (k_2 + k_3)_\mu k_{1\nu} V_3^{\beta\gamma\rho}(k_2, k_3, -k_2 - k_3) \\ &\quad + \frac{A_2}{2d_2} \epsilon_\rho^{\beta\mu\nu} (k_3 + k_1)_\mu k_{2\nu} V_3^{\gamma\alpha\rho}(k_3, k_1, -k_3 - k_1) \\ &\quad + \frac{A_3}{2d_3} \epsilon_\rho^{\gamma\mu\nu} (k_1 + k_2)_\mu k_{3\nu} V_3^{\alpha\beta\rho}(k_1, k_2, -k_1 - k_2), \\ \mathcal{A}_c^{\alpha\beta\gamma} &= \frac{A_1 - A_2}{2(d_1 + d_2)} \epsilon^{\alpha\beta\mu\nu} k_{1\mu} k_{2\nu} (k_3 + 2(k_1 + k_2))^\gamma \\ &\quad + \frac{A_2 - A_3}{2(d_2 + d_3)} \epsilon^{\beta\gamma\mu\nu} k_{2\mu} k_{3\nu} (k_1 + 2(k_2 + k_3))^\alpha \\ &\quad + \frac{A_3 - A_1}{2(d_3 + d_1)} \epsilon^{\gamma\alpha\mu\nu} k_{3\mu} k_{1\nu} (k_2 + 2(k_3 + k_1))^\beta, \end{aligned} \quad (19)$$

where $d_i = k_j \cdot k_k$ with (i, j, k) being cyclic of $(1, 2, 3)$. With three independent momenta k_i we can form three independent angles together with $\theta_{\mu\nu}$, $\alpha_i = k_j \theta k_k / 2$. Then, $A_i = \sin(\alpha_i) \cos(\alpha_j - \alpha_k)$. Note the Bose symmetry is separately satisfied by the three contributions. However the above amplitude does not satisfy the usual QED Ward identity, for example,

$$\left(\mathcal{A}_a^{\alpha\beta\gamma} + \mathcal{A}_b^{\alpha\beta\gamma} + \mathcal{A}_c^{\alpha\beta\gamma} \right) k_{1\alpha} \neq 0; \quad (20)$$

instead, we have,

$$\left(\mathcal{A}_a^{\alpha\beta\gamma} + \mathcal{A}_b^{\alpha\beta\gamma} + \mathcal{A}_c^{\alpha\beta\gamma}\right) k_{1\alpha} k_{2\beta} k_{3\gamma} = 0, \quad (21)$$

which is consistent with the effective non-Abelian nature of $F_{\mu\nu}$. This also implies that we must be careful when computing the physical amplitude squared and summed over polarization states. As we do with the gluons in the usual QCD, we have basically two ways to do so. We may use the unphysical polarization sums for the photons,

$$\sum_{\text{pol}} \epsilon_\mu^* \epsilon_\nu = -g_{\mu\nu}, \quad (22)$$

and then remove the unphysical polarization contribution by subtracting off the ghost contribution to the amplitude squared. The ghost amplitude for the diagram shown in Fig. 1(d) is

$$\mathcal{A}_d = +i16eJ \frac{A_3}{2d_3} \epsilon^{\gamma\mu\nu\rho} k_{1\mu} k_{2\nu} k_{3\rho} \epsilon_{3\gamma}^*. \quad (23)$$

The contribution to be subtracted off, including all cases similar to Fig. 1(d), is,

$$2(16eJ)^2 \left[(A_1)^2 \frac{d_2 d_3}{2d_1} + (A_2)^2 \frac{d_3 d_1}{2d_2} + (A_3)^2 \frac{d_1 d_2}{2d_3} \right], \quad (24)$$

where the factor 2 accounts for the interchange of ghost and anti-ghost. The second way is that we use physical polarization sums for the photons so that only the physical polarization contributions are kept in the amplitude squared. A convenient form is,

$$\sum_{\text{pol}} \epsilon_{i\mu}^* \epsilon_{i\nu} = -g_{\mu\nu} + (k_{i\mu} n_{i\nu} + k_{i\nu} n_{i\mu}) (k_i \cdot n_i)^{-1}, \quad (25)$$

where n_i is an arbitrary vector satisfying $n_i^2 = 0$ and $k_i \cdot n_i \neq 0$. In practice it is most convenient to choose n_i as any of the other two photon momenta.

Now the physical result should not depend on the ways of how to incorporate physical polarizations. But we found that without considering the contribution in Fig. 1(c) originating from the normal π^0 -photon interaction \mathcal{L}_{π^0} there is no way to achieve an identical result in the above two ways. However, including that contribution leads to a unique result which is independent of the ways to do polarization sums and especially of how to choose n_i for the i -th photon. This indicates unambiguously that for consistency of the calculation the π^0 field must transform under $U(1)$ nontrivially as shown in eq. (10) instead of being invariant and that neutral particles must interact with photons in NC-QED.

We are now in a position to compute the decay rate. To make this easier to handle we consider a special case in which $\theta_{0i} = 0$, i.e. we only have space-space noncommutativity. Since Lorentz invariance is lost we have to specify the frame in which the above choice holds. We assume this is the case in the static frame of π^0 ; namely we work in the static frame of π^0 in which $\theta_{0i} = 0$. We warn the reader that the following calculation does not apply to the general case of a moving π^0 or with $\theta_{0i} \neq 0$. The unpolarized differential decay rate is

$$d\Gamma = \frac{1}{3!} \frac{1}{2m_\pi} d\Pi_3 \sum_{\text{pol}} |\mathcal{A}|^2, \quad (26)$$

where

$$\begin{aligned} \sum_{\text{pol}} |\mathcal{A}|^2 &= (16eJ)^2 \frac{2A_3^2}{d_1 d_2 d_3} \left[2^{-4} m_\pi^8 + (d_1^4 + d_2^4 + d_3^4) \right], \\ d\Pi_3 &= \Pi_{i=1}^3 \left[\frac{d^3 \vec{k}_i}{(2\pi)^3 2|\vec{k}_i|} \right] (2\pi)^4 \delta^4(p - k_1 - k_2 - k_3). \end{aligned} \quad (27)$$

The expression for $\sum_{\text{pol}} |\mathcal{A}|^2$ simplifies considerably in the current case because we actually have only one independent angle made of k_i and $\theta_{\mu\nu}$ so that $A_1 = A_2 = A_3$ and the \mathcal{A}_c term disappears.

Special attention should be paid to the phase space calculation due to the same reason of Lorentz noninvariance. We might calculate in two steps as usual; first we integrate over \vec{k}_1 and \vec{k}_2 in the static frame of $\vec{k}_1 + \vec{k}_2$, then we move to the static frame of π^0 to finish the remaining integration. But this is simply wrong without modifying correspondingly the constant matrix $\theta_{\mu\nu}$ from one frame to another. Actually, we would otherwise obtain a vanishing result for the above specified case. It turns out that it is much more convenient to fix the frame from the very beginning and work out kinematics in terms of Euler angles. Without loss of generality we assume $\theta_{12} = -\theta_{21} = \theta$ and others vanishing, i.e. the ‘magnetic’ constant field $B^i = \epsilon^{ijk} \theta^{jk}/2$ points in the z direction. We use the polar and azimuthal angles β and γ to define the normal direction of the event plane, and another azimuthal angle α to fix the absolute direction in the event plane. For example we may use as a reference direction for this purpose the intersection between the event plane and the plane spanned by the normal of the event plane and the z axis although our final result is independent of α . Finishing part of integration using the δ^4 function, we have

$$d\Pi_3 = 2^{-8} \pi^{-5} d\omega_1 d\omega_2 d\alpha d\cos\beta d\gamma, \quad (28)$$

with $\omega_i = |\vec{k}_i|$. For a given $0 \leq \omega_1 \leq m_\pi/2$, we have $(m_\pi - 2\omega_1)/2 \leq \omega_2 \leq m_\pi/2$. The kinematic quantities are computed below:

$$\begin{aligned} d_i &= m_\pi(m_\pi - 2\omega_i)/2, \quad \sum_{i=1}^3 \omega_i = m_\pi/2, \\ \alpha_i &= \sqrt{m_\pi(m_\pi - 2\omega_1)(m_\pi - 2\omega_2)(m_\pi - 2\omega_3)(\cos \beta)\theta}/4 \\ &= \sqrt{d_1 d_2 d_3} \frac{\theta \cos \beta}{\sqrt{2} m_\pi}. \end{aligned} \quad (29)$$

Since $|\theta| m_\pi^2 \ll 1$, we have $2A_3^2/(d_1 d_2 d_3) \approx (\theta m_\pi^2)^2 \cos^2 \beta / m_\pi^6$. Completing integration over $\omega_{1,2}$, α and γ results in the following differential decay rate,

$$\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \beta} = \frac{3}{2} \cos^2 \beta, \quad \Gamma = \frac{\alpha^3 N_c^2 m_\pi^3}{2^9 3^3 5 \pi^4 f_\pi^2} (\theta m_\pi^2)^2. \quad (30)$$

Thus, the decay occurs dominantly in the plane which is perpendicular to the ‘magnetic’ constant field while it is forbidden in any plane which is parallel to the field. This is a particular feature of the star product coupling among fields. The simplicity of the above distribution also has its origin in the much simpler gauge invariant structures shown in eq. (19) as compared to the case of the standard model. Therefore the differential distributions in the two cases are completely different.

We note that the $\pi^0 \rightarrow 2\gamma$ decay is also modified, see eq. (17). This modification is generally very small and vanishes in the particular case specified above. So, the branching ratio is,

$$\text{Br}(\pi^0 \rightarrow 3\gamma) = \frac{\Gamma(\pi^0 \rightarrow 3\gamma)}{\Gamma(\pi^0 \rightarrow 2\gamma)} = \frac{\alpha}{120\pi} (\theta m_\pi^2)^2. \quad (31)$$

For a noncommutativity of order $\sqrt{|\theta|} = 1 \text{ TeV}^{-1}$, we have,

$$\text{Br}(\pi^0 \rightarrow 3\gamma) = 6.4 \cdot 10^{-21}. \quad (32)$$

This result is larger than an estimate in the standard model by many orders of magnitude [5], but still far below the current experimental upper bound of $3.1 \cdot 10^{-8}$ [9]. Further, a much smaller branching ratio will result if some very stringent limits on $|\theta|$ are applied. We also mention in passing that \mathcal{L}_{NCA} also implies a C-conserving four photon decay of π^0 . But it is less interesting and cannot exceed the standard model process that occurs at one loop order in chiral perturbation theory involving anomaly, with a branching ratio of order $5.5 \cdot 10^{-17}$ [10].

We have shown how a simple and reasonable generalization of the anomalous π^0 -photon interaction can lead to the C-violating three photon decay of the π^0 in NCQED.

We demonstrated explicitly that for such a consideration to be physically self-consistent it is mandatory to treat the electrically neutral photon and π^0 on the same footing in the sense that they transform under $U(1)$ as if they were in the adjoint representation of a non-Abelian gauge theory. The neutral particles thus also enjoy electromagnetic interactions. This is reminiscent of the wisdom in the usual field theory that one must keep all possible interactions that are consistent with symmetries for the theory to be renormalizable. We are thus inclined to believe that the above conclusion should be a general result in noncommutative field theories. Phenomenologically the branching ratio we obtained is generally much larger than the estimated result in the standard model. In the case of space-space noncommutativity and with a static π^0 , the decay distribution has a simple characteristic of being preferred in a direction specified by the θ parameter, which would be quite helpful in experimental identification if there were any chance to detect such a small branching ratio.

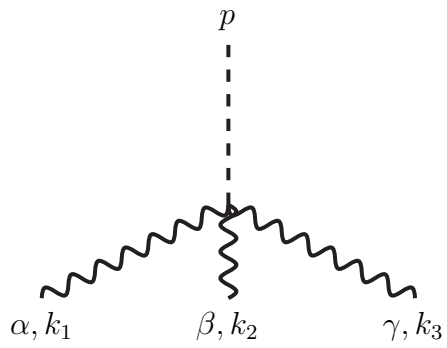
Acknowledgements We are grateful to Klaus Sibold for many encouraging and helpful discussions and for carefully reading the manuscript. H.G. enjoyed the stay at ITP, Universität Leipzig where part of work was done.

References

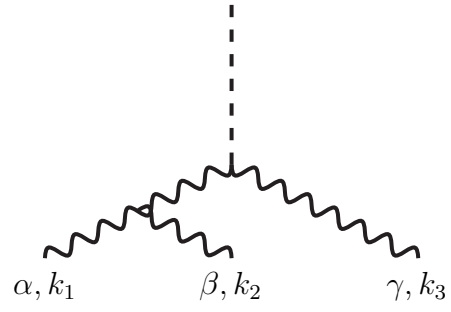
- [1] A. Connes, M. R. Douglas, A. Schwarz, JHEP 9802 (1998) 003; M. R. Douglas, C. Hull, JHEP 9802 (1998) 008; N. Seiberg, E. Witten, JHEP 9909 (1999) 032.
- [2] C. P. Martin, D. Sanchez-Ruiz, Phys. Rev. Lett. 83 (1999) 476; T. Krajewski, R. Wulkenhaar, Int. J. Mod. Phys. A 15 (2000) 1011; M. M. Sheikh-Jabbari, JHEP 9906 (1999) 015; M. Hayakawa, Phys. Lett. B 478 (2000) 394; hep-th/9912167; A. Matusis, L. Susskind, N. Toumbas, JHEP 0012 (2000) 002.
- [3] I. Mocioiu, M. Pospelov and R. Roiban, Phys. Lett. B 489 (2000) 390; M. Chaichian, M. M. Sheikh-Jabbari, A. Tureanu, Phys. Rev. Lett. 86 (2001) 2716.
- [4] H. Arfaei, M. H. Yavartanoo, hep-ph/0010244; J. L. Hewett, F. J. Petriello, T. G. Rizzo, hep-ph/0010354; P. Mathews, Phys. Rev. D 63 (2001) 075007; S. Baek, D. K. Ghosh, X.-G. He, W.-Y. P. Hwang, Phys. Rev. D 64 (2001) 056001.
- [5] D. A. Dicus, Phys. Rev. D 12 (1975) 2133.
- [6] M. M. Sheikh-Jabbari, Phys. Rev. Lett. 84 (2000) 5265.
- [7] J. Wess and B. Zumino, Phys. Lett. 37 B (1971) 95; E. Witten, Nucl. Phys. B 223 (1983) 422; see also, e.g., D. J. Gross and R. Jackiw, Phys. Rev. D 6 (1972) 477.
- [8] F. Ardalan, N. Sadooghi, Int. J. Mod. Phys. A 16 (2001) 3151 and hep-th/0009233; J. M. Gracia-Bondia, C. P. Martin, Phys. Lett. B 479 (2000) 321; C. P. Martin, hep-th/0008126.
- [9] J. McDonough et al, Phys. Rev. D 38 (1988) 2121.
- [10] R. L. Schult, B.-L. Young, Phys. Rev. D 6 (1972) 1988; Y. Liao, Phys. Rev. D 57 (1998) 1573; for a much larger branching ratio based on the purely electromagnetic photon splitting, see, E. L. Bratkovskaya, E. A. Kuraev and Z. K. Silagadze, Phys. Lett. B 359 (1995) 217.

Figure Captions

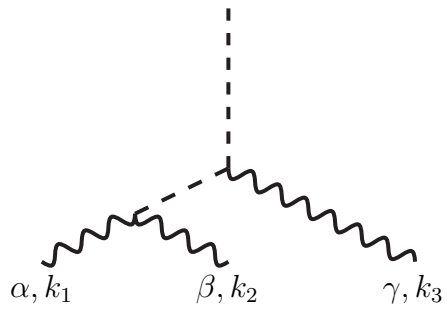
Fig. 1 Feynman diagrams for the decay $\pi^0 \rightarrow 3\gamma$ involving the contact interaction (*a*), the NCQED three photon vertex (*b*), the normal NCQED π^0 -photon vertex (*c*), and a ghost-anti-ghost pair in the final state (*d*). The dashed, wavy and solid lines stand for the pion, photon and ghost fields respectively. Diagrams obtained by permutation are not shown.



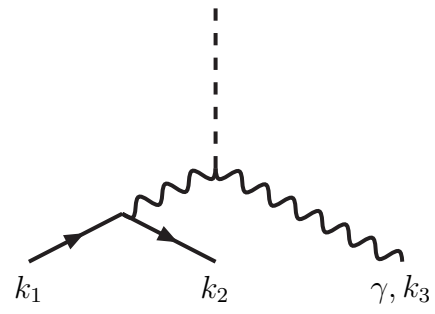
(a)



(b)



(c)



(d)

Figure 1